Recursively stacking items with variable height in a tube packing application

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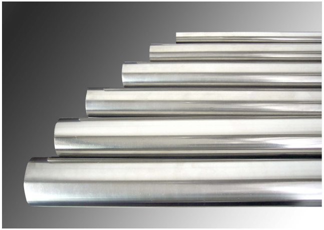
Tube packing



Semifluids

- Materials that share properties both with solids and with fluids
- Related notions:
  - fluidized bed—particles whose behaviour is fluid, due to the introduction of a pressurized stream
  - non-Newtonian fluid—e.g., toothpaste: has a shape, but under pressure flows like a liquid

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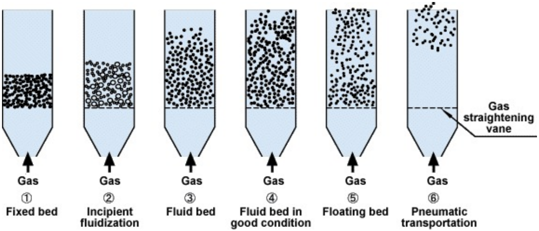

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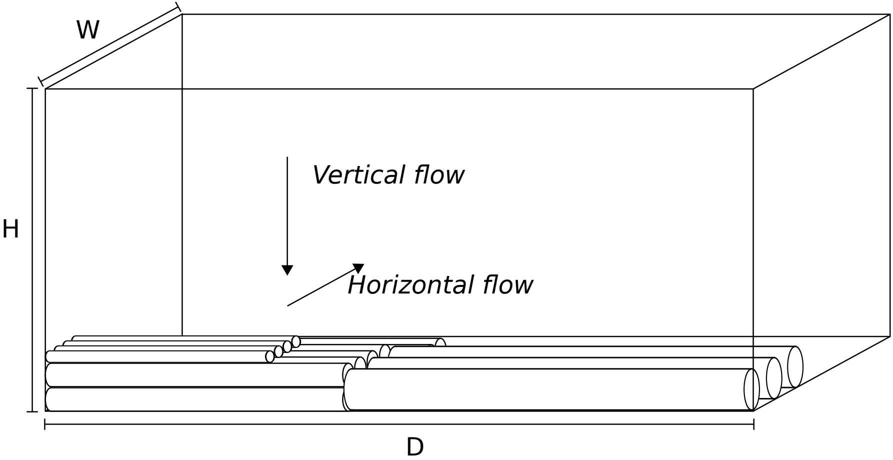
Problem definition

Tube packing problem

Given multiple sets of tubes, each set is homogeneous in regards to tube length and diameter, find a packing that maximises profit.

- All tubes must flow in the same direction.
- Tubes can only be stacked on tubes of longer length.

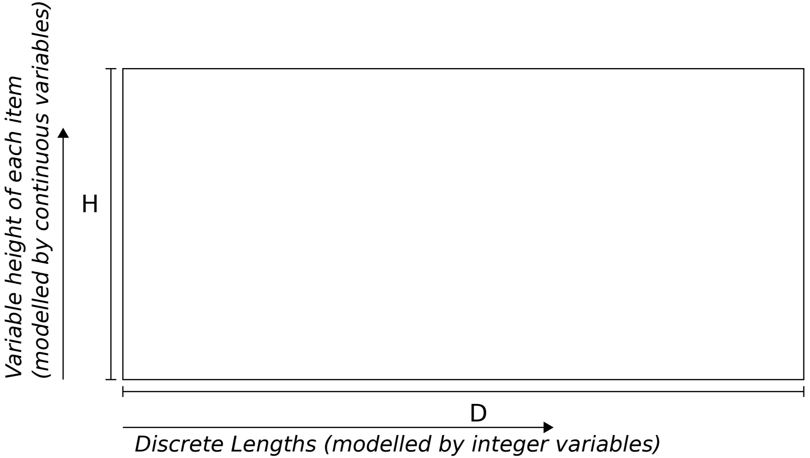
Packing



Assumptions

- There is complete horizontal flow
- The vertical level filled can be represented by a continuous variable.

Tube packing problem



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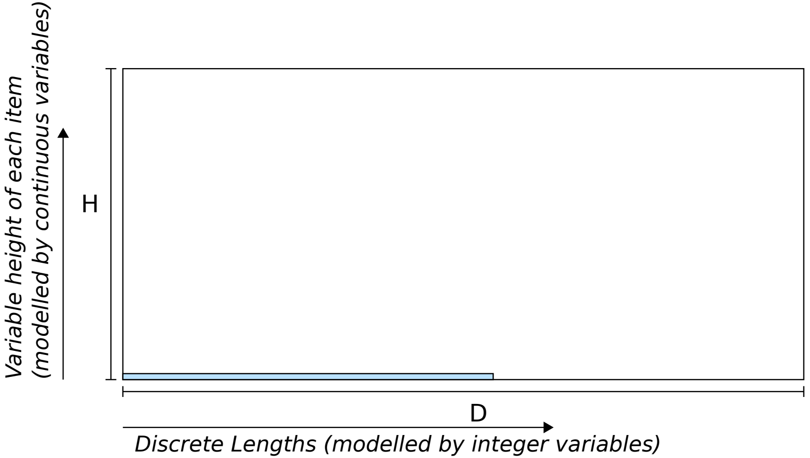

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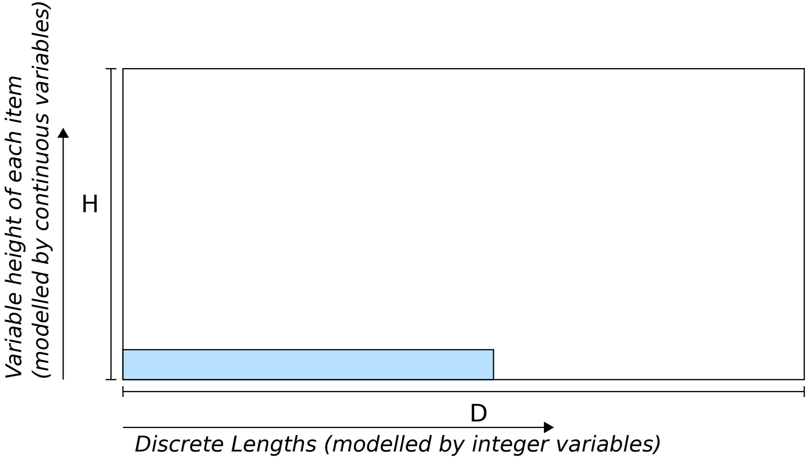

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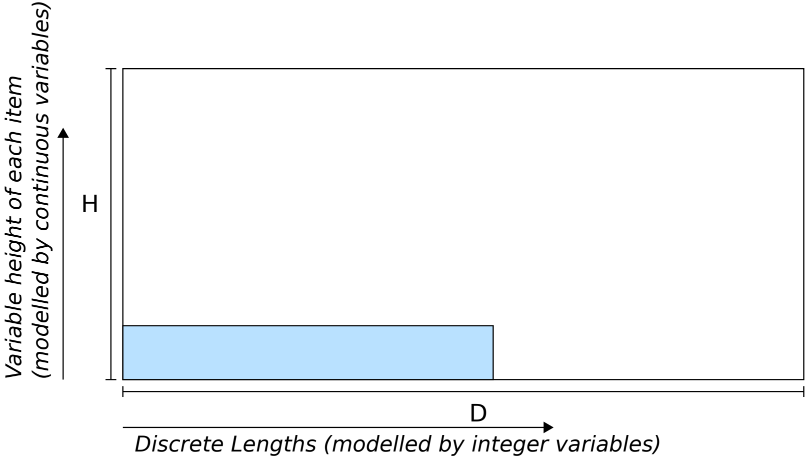

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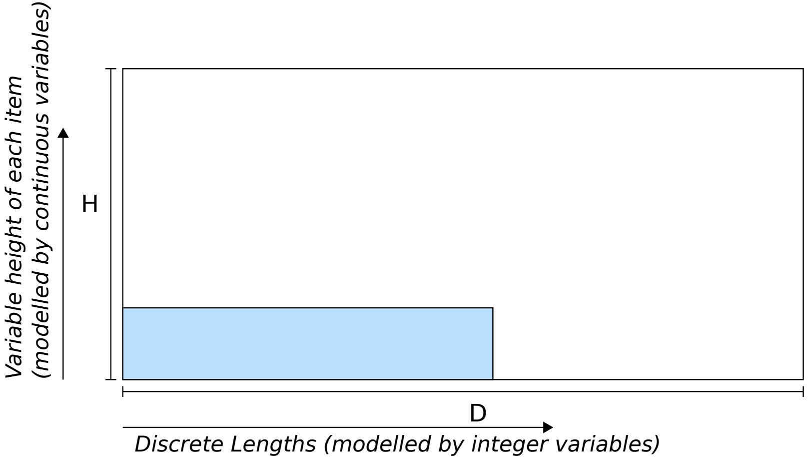

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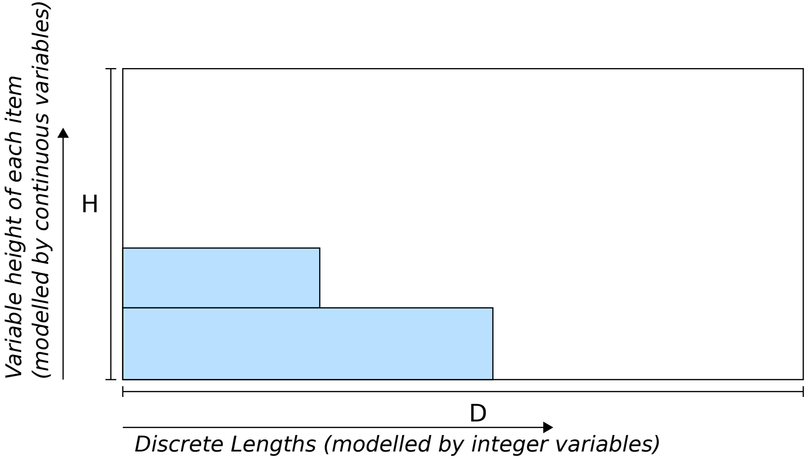

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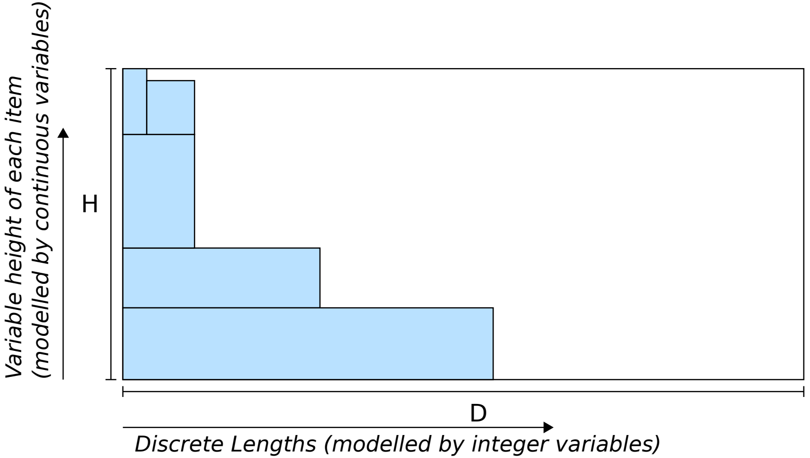

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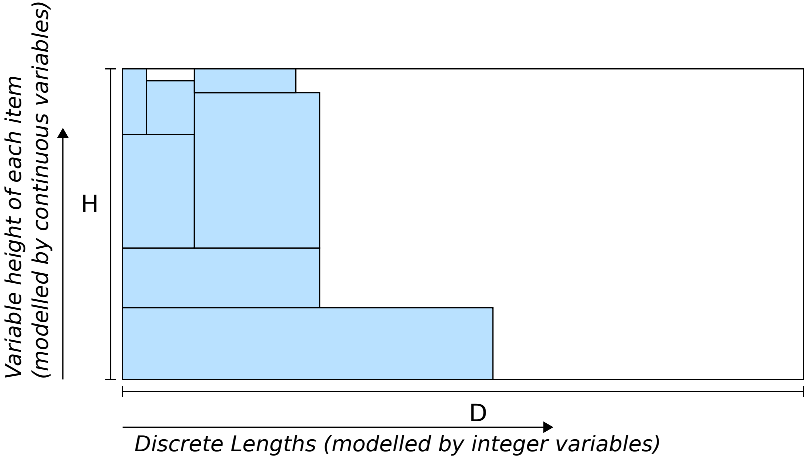

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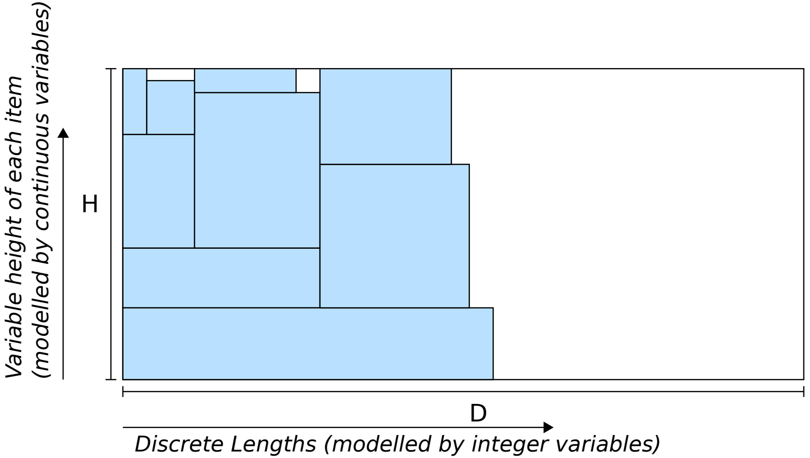

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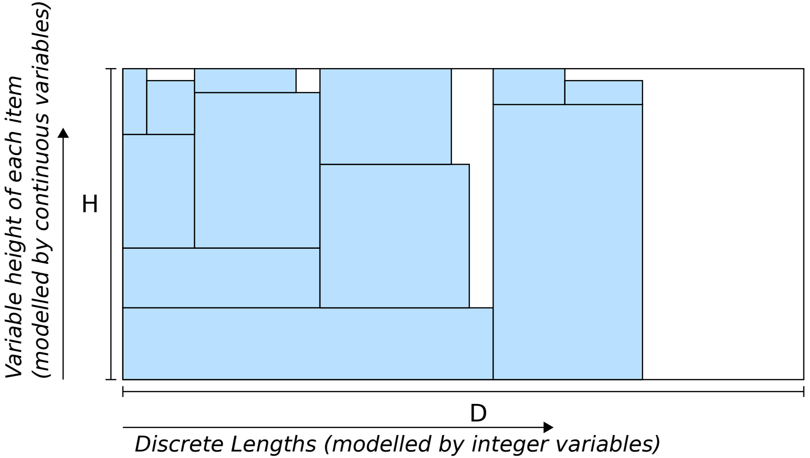

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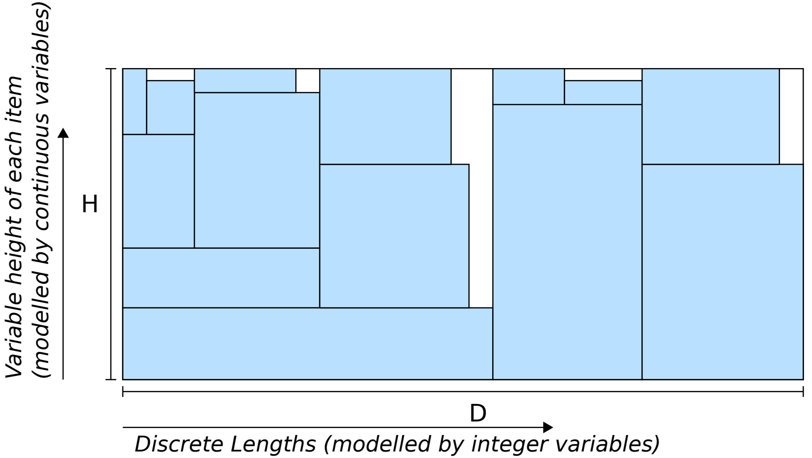

Similar problems

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Ignoring the variable height

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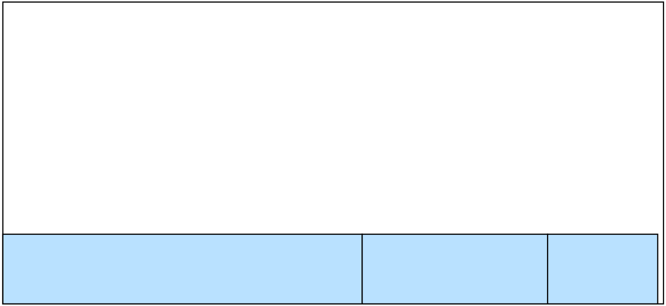

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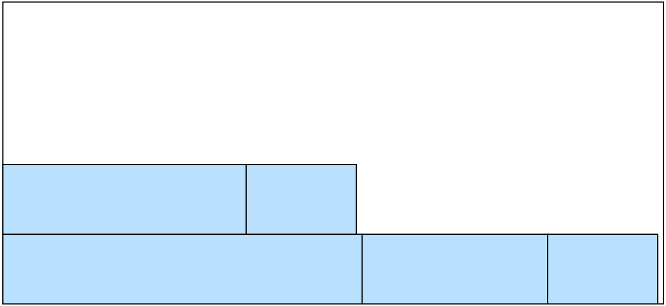

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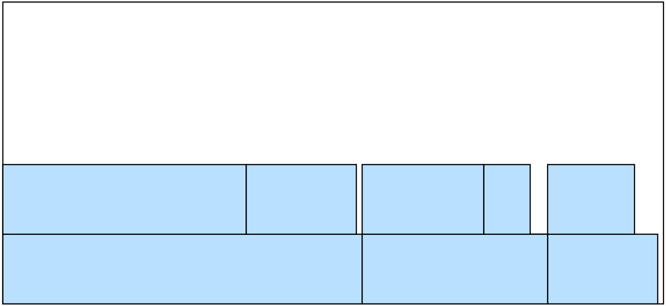

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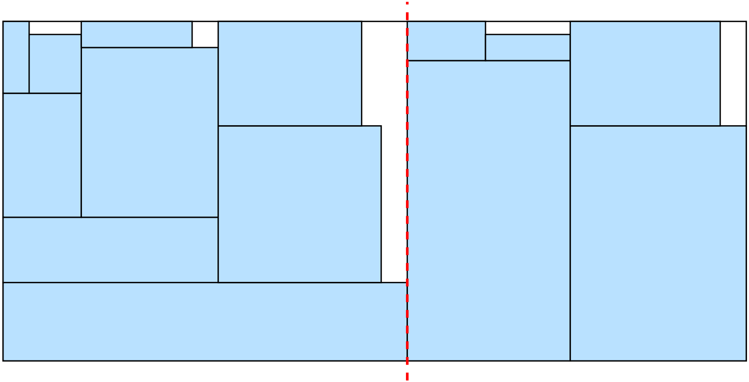
Consider guillotine cuts



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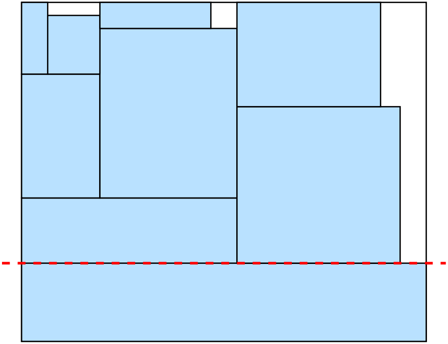

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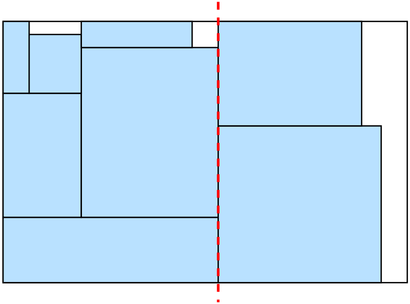

Knapsack problem

- Recursive structure.
- The heights of the items are not fixed.
- Items are splittable.

Guillotine cutting stock problem

- Must be alternate between vertical and horizontal cuts.
- Horizontal cuts must correspond with the lengths of tubes.
- Vertical cuts define the fraction of an item that is used.

Graph theory definition

Given

- a directed graph $G = (V, A)$ with a single source s,
- n groups of vertices $V = \cup_{i=1}^{n} V_i$, $V_i \cap V_j = \emptyset, \forall i, j, i \neq j$,
- the set of all paths $\mathcal{P}$ from s,
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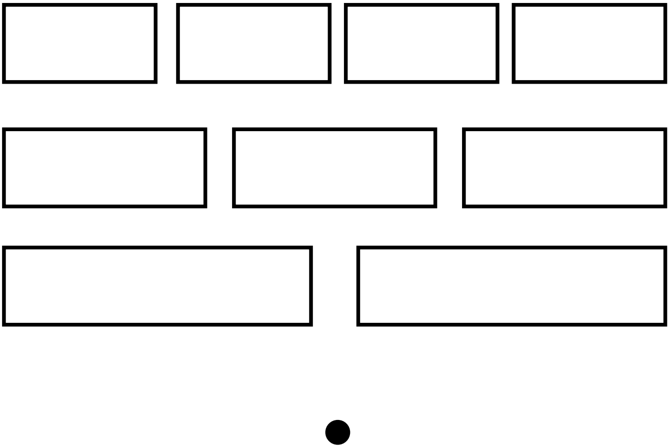
Example graph

Consider a tube packing problem with three items of length 1/2, 1/3, 1/4.



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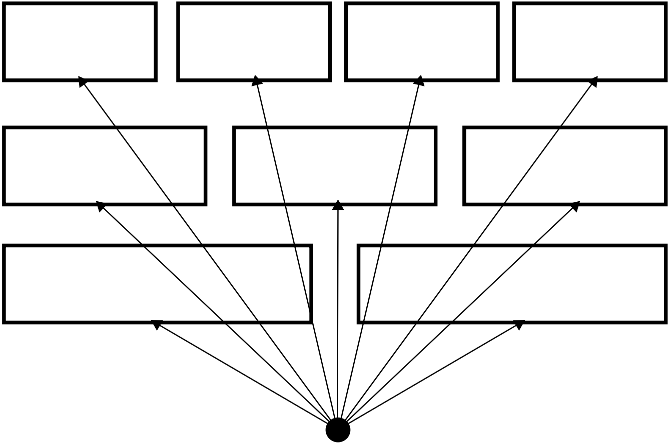

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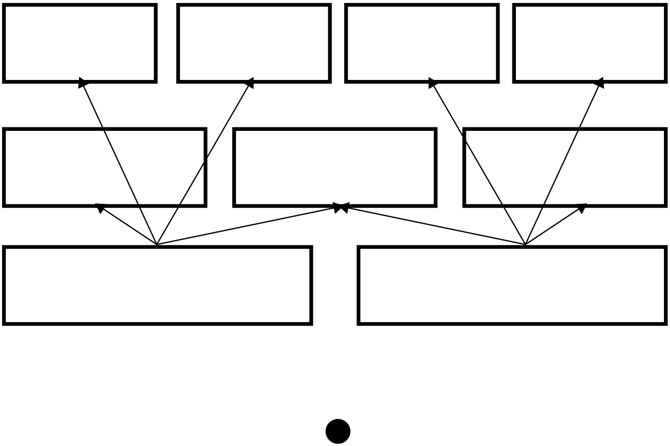

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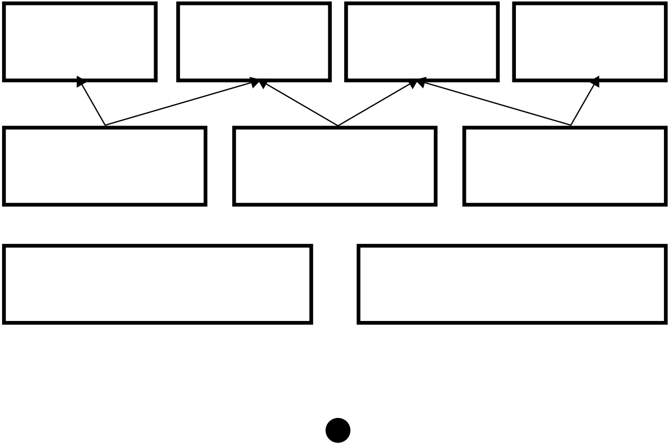

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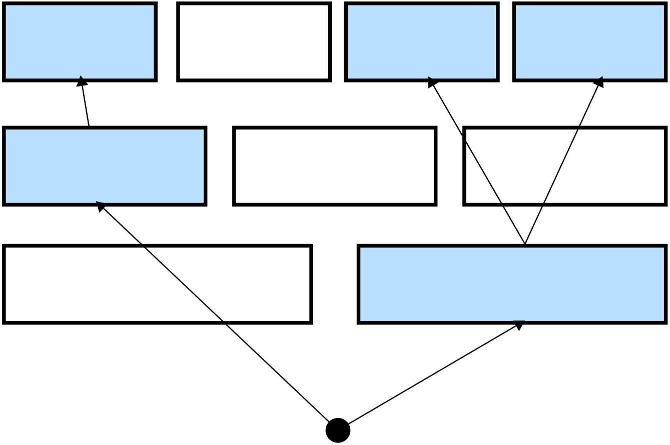

Problem definitions

Data

- directed graph $G = (V, A)$
- t_i is the item type of each vertex
- for each item n, the corresponding length l_n and possible height h_n
- container dimensions D, H

Variables

- $y_{ij} = 1$ if arc (i, j) is used (i.e, item t_j is packed on top of t_i)
- $x_{ij} \in [0, 1] \rightarrow$ fraction of item t_j used in (i, j)
- $p_i \in [0, H] \rightarrow$ potential of each vertex i, with $p_s = 0$

Semifluid packing problem (SfPP) - formulation

$$\max \sum_{n \in N} \sum_{\substack{(i,j) \in A | \\ t_j = n}} w_n x_{ij}, \tag{1}$$

$$\sum_{\substack{j \in V | \\ (s,j) \in A}} l_{t_j} y_{sj} \leq D, \tag{2}$$

$$\sum_{\substack{k \in V | \\ (j,k) \in A}} l_{t_k} y_{jk} \leq l_{t_j} y_{ij} \qquad \forall j \in V, (i,j) \in A, \tag{3}$$

$$x_{ij} \leq y_{ij} \qquad \forall (i,j) \in A, \tag{4}$$

$$p_j \geq p_i + x_{ij} h_{t_j} - H(1 - y_{ij}) \qquad \forall (i,j) \in A, \tag{5}$$

$$\sum_{\substack{(i,j) \in A | \\ t_j = n}} x_{ij} \leq 1 \qquad \forall n \in N, \tag{6}$$

$$y_{ij} \in \{0,1\},\ x_{ij} \in [0,1],\ p_i \in [0,H]. \tag{7}$$

Difficulties

- Graph is very large with modest input.
- Size of the problem formulation is directly related to the graph.
  - Large number of variables and constraints
- Complex relationship between continuous and discrete variables.

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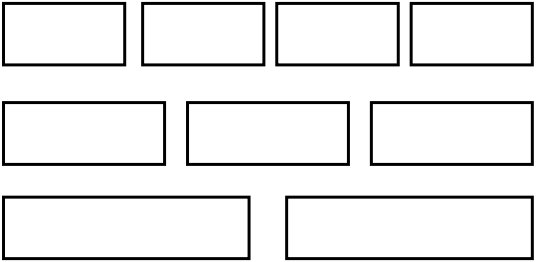
Potential solution: Decomposition by variables or constraints.

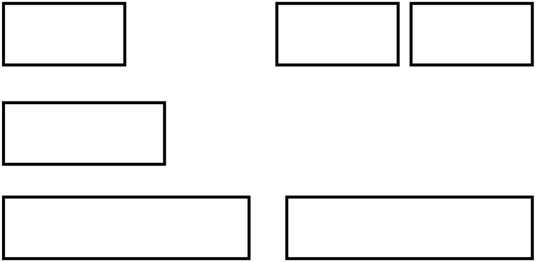
Benders' decomposition

- Simple separation of continuous and discrete variables—classical Benders' decomposition.



Benders' decomposition

- Simple separation of continuous and discrete variables—classical Benders' decomposition.
- Master problem identifies packing structure.



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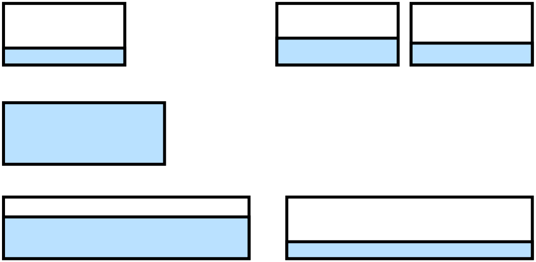
- Subproblem sets the amount of each item that is pack on the given structure.



Data

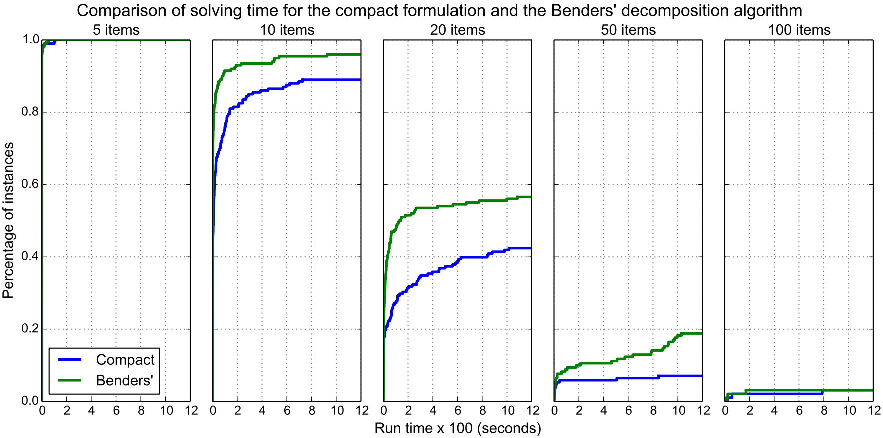
- Problem is derived from a practical application; however, instances are small.
- All test instances are generated for the computational experiments.

Test instances

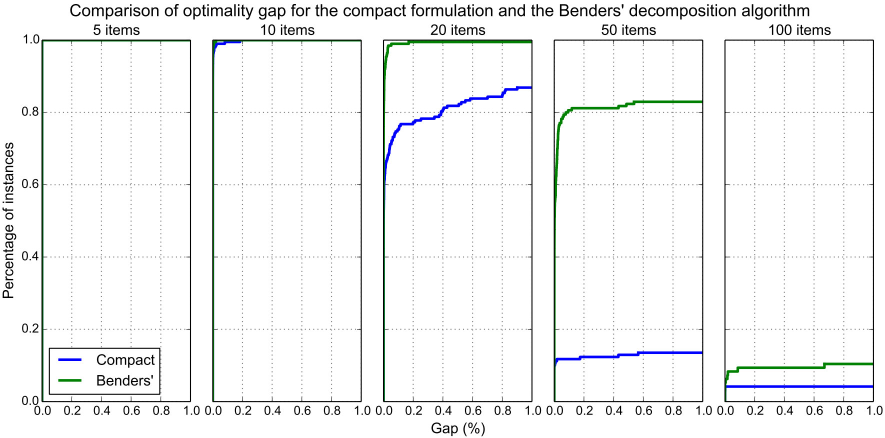
- Easy instances: all items can be packed into the container.
- Hard instances: the volume of the available items is either
  - 100% of the container volume,
  - 150% of the container volume.
- Instances with 5, 10, 20, 50 and 100 semifluids.
- 200 instances with each combination—total 3000 instances.

Computational results - Easy instances

Time



Gap


Comparison of optimality gap for the compact formulation and the Benders' decomposition algorithm
5 items
10 items
20 items
50 items
100 items
Percentage of instances
Gap (%)
0.0
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0.4
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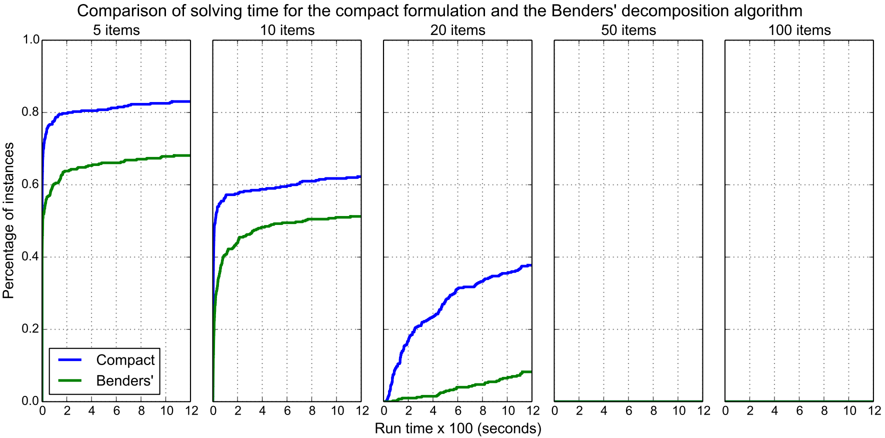
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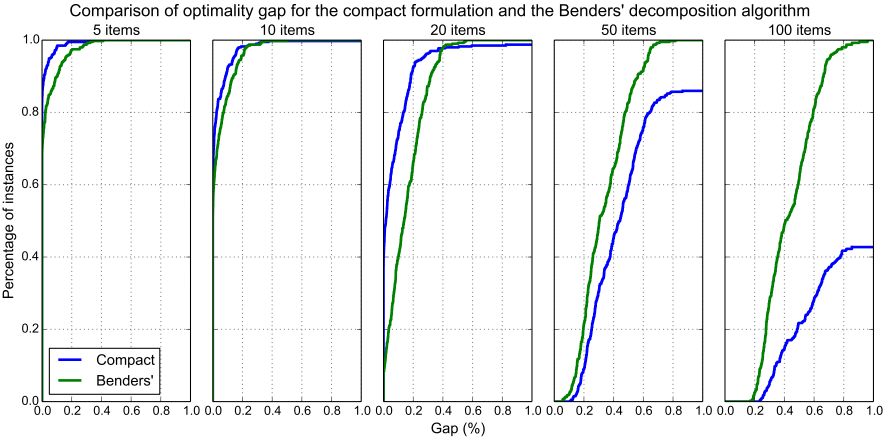
0 2 4 6 8 10 12

Computational results - Hard instances

Gap



Conclusion

- Presented a model for packing tubes into containers based upon the concept of semifluids.
- Provided a mathematical formulation for the semifluid packing problem.
- Developed a compact MIP formulation based upon the graph structure.
- Compared the effectiveness of the compact formulation and the Benders' decomposition algorithm.