Addressing degeneracy in the dual simplex algorithm using a decompositon approach

Ambros Gleixner, Stephen J Maher, Matthias Miltenberger

Zuse Institute Berlin
Berlin, Germany

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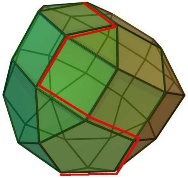
@sj_maher

#ISMP2015

Simplex method - effective LP solving

- Effective approach to solve linear programming problems.
- Fundamental for solving mixed integer programs.
- Algorithm developments have greatly improved effectiveness.
  - Linear algebra,
  - pricing and pivoting rules,
  - presolving techniques,
  - ...



Motivation

Mixed integer programming

- Significant part of mixed integer programming solvers.
- Large proportion of solving time.
- Degeneracy still impacts effectiveness of simplex based solvers.

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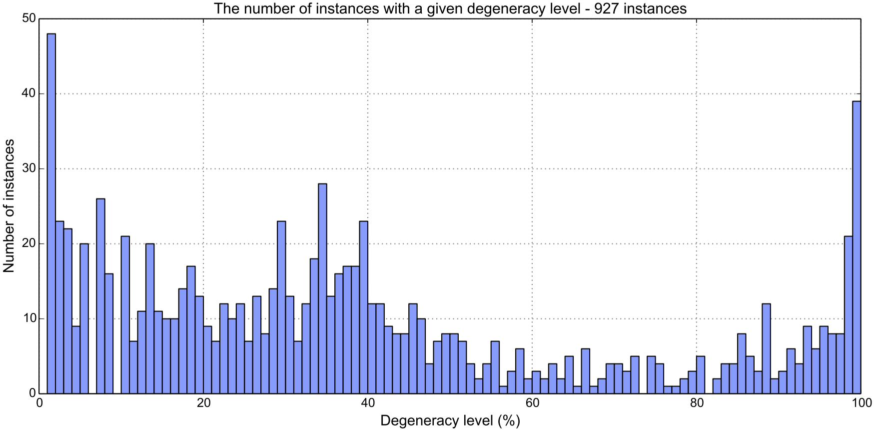
Goal: Reduce the degeneracy encountered in the simplex method to improve algorithm performance.

Outline

Prevalence of dual degeneracy



Collection of 1,226 LP instances - 299 have a degeneracy level less than 1%.

Approaches to address degeneracy

Primal degeneracy

Improved Primal Simplex - Elhallaoui et al. (2011)

- Developed with a focus on set partitioning problems.
- Decomposes problem to contain only non-degenerate primal variables.
- Further developments have resulted in an effective solution approach.

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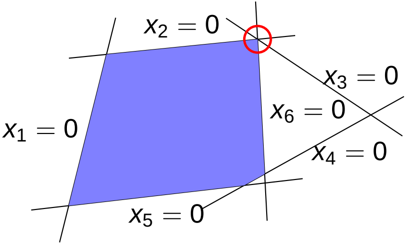

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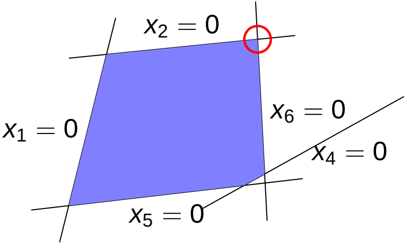


Approaches to address degeneracy

Dual degeneracy

- Perturbation
  - Modification of the objective function coefficients
- Bound flipping ratio test
  - Changing variable from one bound to another without affecting dual feasibility.
  - Larger steps are afforded.

Outline

Introduction

Decomposition approach for the dual simplex

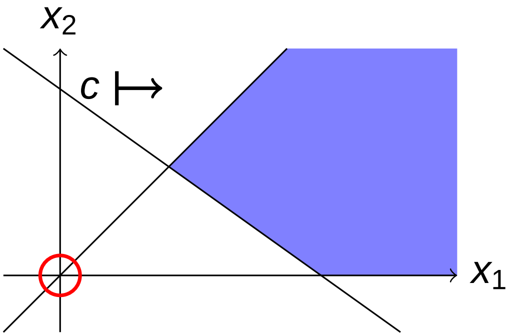
Results

Conclusion

Reducing dual degeneracy

Consider $P = \max\{c'x : Ax \leq b\}$ and let x^* be a basic solution.

Degeneracy occurs if more than m primal variables have a reduced cost of zero.



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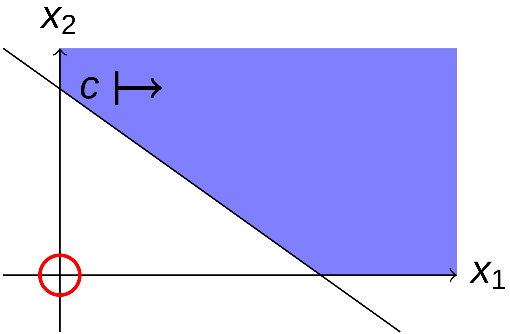

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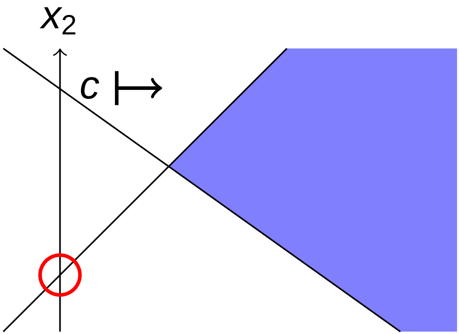

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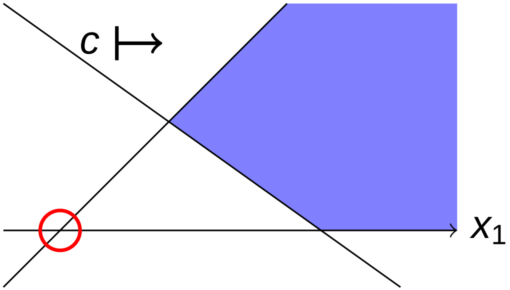

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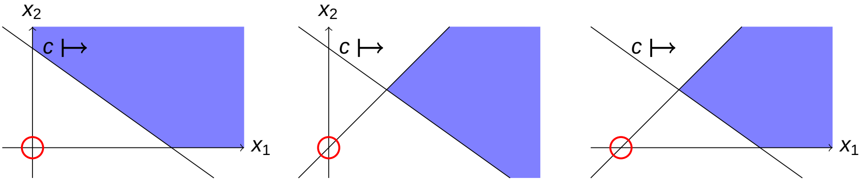
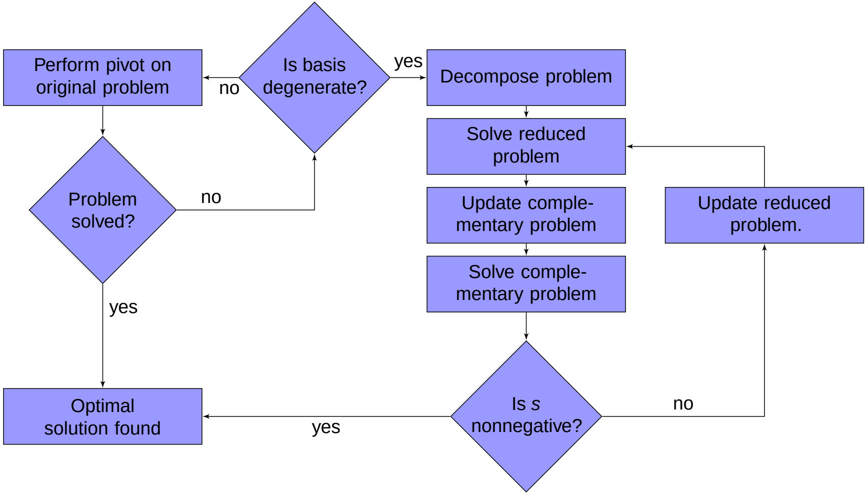
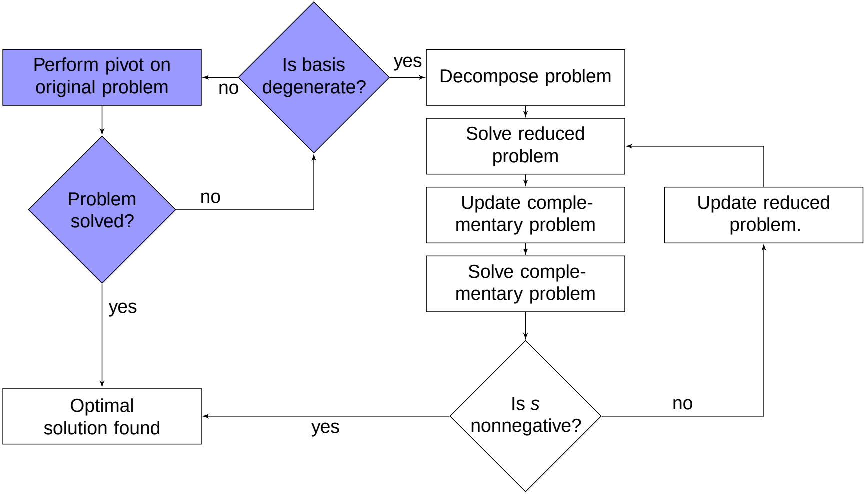


Algorithm



Perform pivot on original problem
no
Is basis degenerate?
yes
Decompose problem
Solve reduced problem
Problem solved?
no
Update comple-mentary problem
Update reduced problem.
Solve comple-mentary problem
yes
Optimal solution found
yes
Is s nonnegative?
no

Algorithm


Perform pivot on original problem
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Decompose problem
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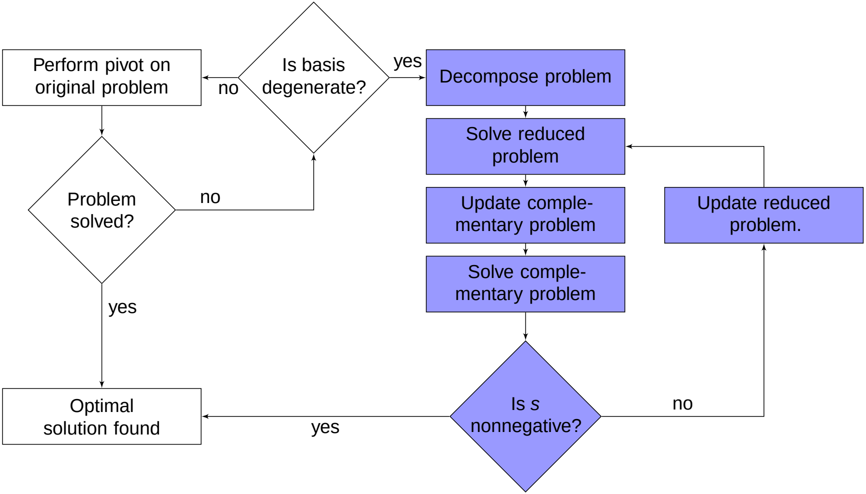

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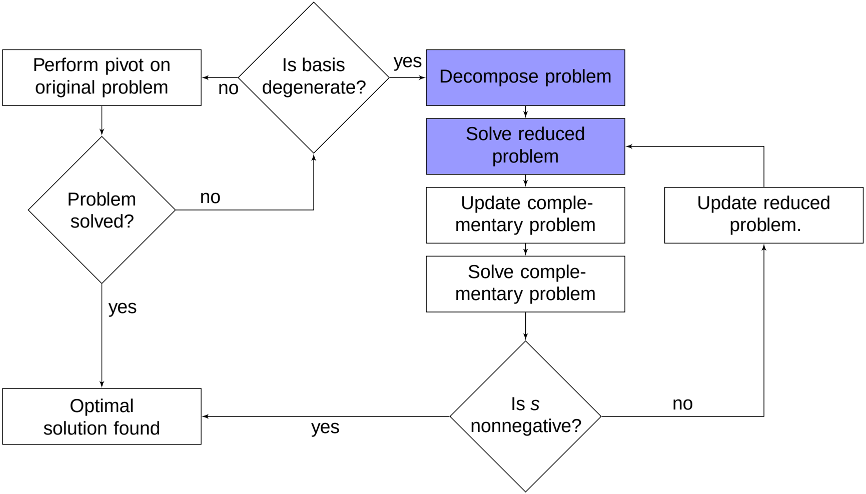
Decomposition



Decomposition

Given $P = \max\{c'x : Ax \leq b\}$, and a row form basis B ($Q = B^{-1}$).

- Partition the indices of B by the dual multipliers.
  - $\mathcal{P}$ where $y_i > 0$ and $\mathcal{N}$ where $y_i = 0$.
- Reorder basis: $Q = (Q_{\cdot\mathcal{P}} | Q_{\cdot\mathcal{N}})$.
- Substitute $x = Q\bar{x}$, where $x = Q_{\cdot\mathcal{P}}\bar{x}_{\mathcal{P}} + Q_{\cdot\mathcal{N}}\bar{x}_{\mathcal{N}}$.
- Compatible rows identified by $A_{i\cdot}Q_{\cdot\mathcal{N}} = 0$, $i \in \{1, \ldots, m\}$.

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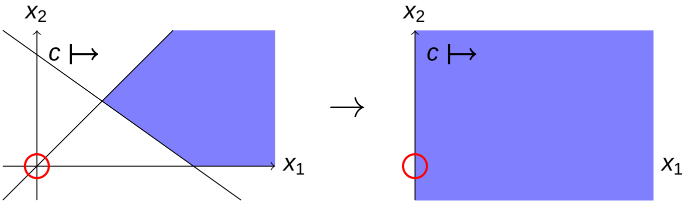
$$\begin{aligned} \max \quad & (c'Q_{\cdot\mathcal{P}})\bar{x}_{\mathcal{P}} + (c'Q_{\cdot\mathcal{N}})\bar{x}_{\mathcal{N}} \\ \text{subject to} \quad & A_{\mathcal{C}\cdot}Q_{\cdot\mathcal{P}}\bar{x}_{\mathcal{P}} + A_{\mathcal{C}\cdot}Q_{\cdot\mathcal{N}}\bar{x}_{\mathcal{N}} \leq b_{\mathcal{C}}, \\ & A_{\mathcal{I}\cdot}Q_{\cdot\mathcal{P}}\bar{x}_{\mathcal{P}} + A_{\mathcal{I}\cdot}Q_{\cdot\mathcal{N}}\bar{x}_{\mathcal{N}} \leq b_{\mathcal{I}}. \end{aligned}$$

Reduced problem



The reduced costs are: $c'Q = y_{\mathcal{B}}$.

Given $c'Q_{\cdot\mathcal{N}} = y_{\mathcal{N}} = 0$ and $A_{\mathcal{C}\cdot}Q_{\cdot\mathcal{N}} = 0$.

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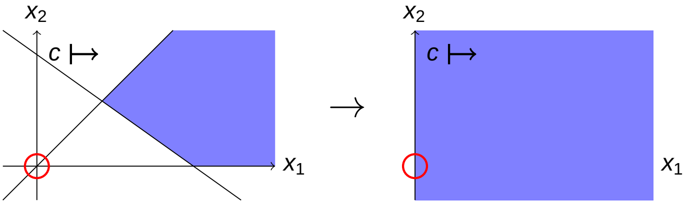

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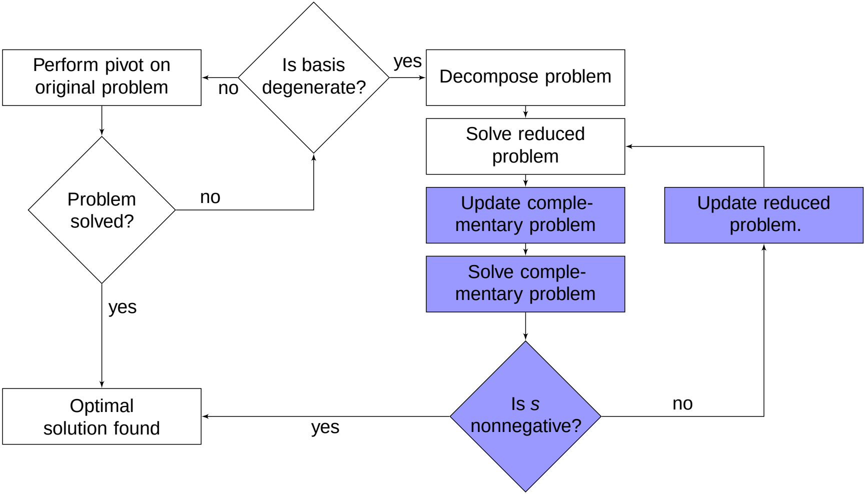
Complementary problem



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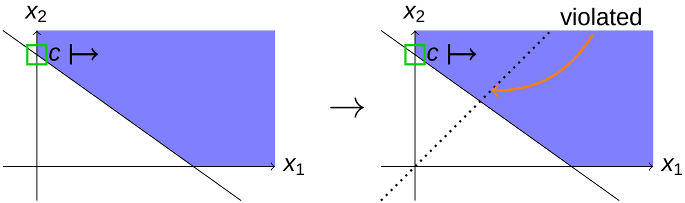
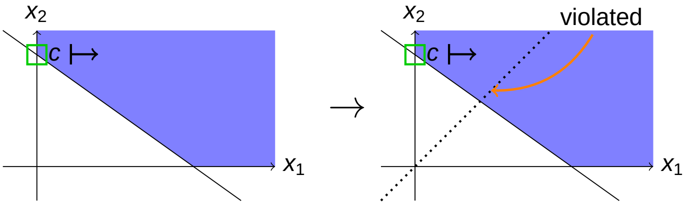


Know: Constraints currently at bounds.

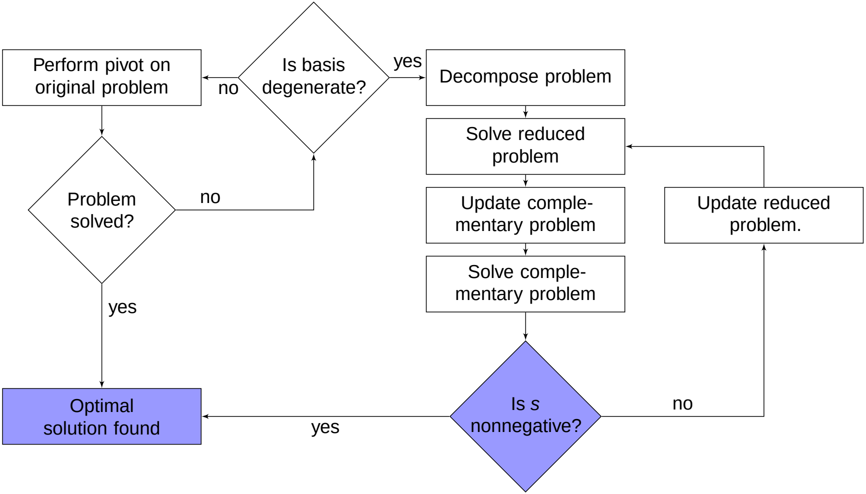
Complementary problem



Know: Constraints currently at bounds.

$$
\begin{aligned}
\max \quad & s \\
\text{subject to} \quad & A_{i\cdot}x = b_i \quad \forall i \in \mathcal{P}^*, \\
& A_{i\cdot}x + s \leq b_i \quad \forall i \in \mathcal{I}^*, \\
& s \in \mathbb{R}.
\end{aligned}
$$

Termination



Implementation

Implemented within SoPlex

Simplex based LP solver developed at ZIB.
Available from `soplex.zib.de`

- Row representation available in SoPlex is necessary.
- Initial algorithm for original, reduced and complementary problems is the dual simplex.
- No Phase 1 - degeneracy detected by subtracting bound shifts.

Complementary problem solved in the dual representation.

- Violated constraints are detected using a ratio test - comparing dual solutions from the reduced and complementary problems.

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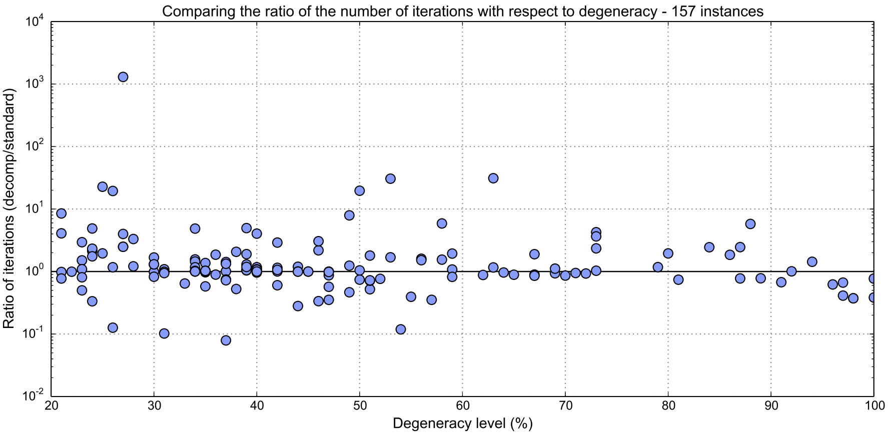
Test instances

- Set of instances collected from numerous LP and MIP benchmark testsets - total of 1,226
- Only considered those solved in one hour using dual algorithm of SoPlex in row representation and by the decomposition algorithm.
- Removed instances solved by SoPlex in less than 0.05 seconds and displayed average degeneracy less than 20%.
- Final testset contained 157 instances.

Iterations change



Conclusion

- Presented a decomposition approach designed to reduce the impact of degeneracy.
- Discussed the employed solution algorithm and the implementation in the freely available code of SoPlex (`soplex.zib.de`).
- Demonstrated the potential of this approach to reduce the iterations of the dual simplex method.

The SCIP Optimization Suite

LP

- SoPlex: a new exact LP solver → TB07 Ambros Gleixner
- The Impact of LP on . . . MIP Solvers → WB16 Matthias Miltenberger

MIP

- Recent Advances in the SCIP Opt. Suite for Solving MIPs → MD16 Gregor Hendel
- Reoptimization Techniques in MIP Solvers → WB01 Jakob Witzig
- Recent Branching Improvements for MIP . . . → WC07 Gerald Gamrath
- Branch-and-Cut for . . . Overlapping SOS1 Constraints → WC07 Tobias Fischer

MINLP

- Solving MISDPs for Robust Truss Topology Design → TB09 Tristan Gally
- (Pre-)Solving Nonlinear Pseudo-Boolean . . . Problems → WC07 Michael Winkler
- Recent Advances in Solving MINLPs with SCIP → ThC24 Benjamin Mueller
- On Outer-Approximations of Convex Regions → ThC24 Felipe Serrano

Parallel & Multiobjective

- PolySCIP – A Solver for Multi-Criteria MIPs → WD23 Sebastian Schenker
- How to Run a Customized SCIP Solver on . . . 80,000 Cores → ThC15 Yuji Shinano