

# The single day aircraft maintenance routing problem

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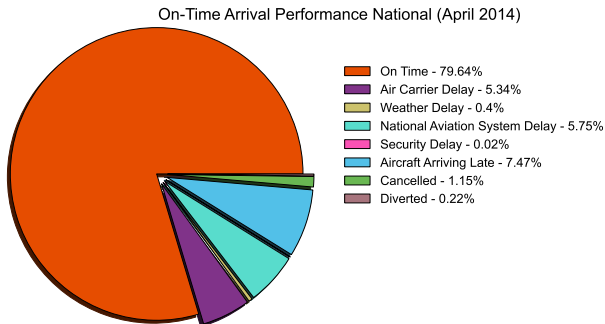
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## The real world is uncertain

- It is almost impossible to achieve 100% on-time performance.
- Events are difficult to predict, effects are unknown.



**Figure :** On-Time Arrival Performance National (April 2014). Source: Bureau of Transportation Statistics, Airline Service Quality Performance 234

# Effect of disruptions

## Short Term

- Requires recovery actions, such as flight delays and cancellations.
- Increase in operational costs.

## Longer Term

- Planned crew pairings and aircraft routings for subsequent days are affected.
- As a result, crew may be unable to return home or aircraft unable to receive maintenance.

# Outline

- 1 Introduction
  - Aircraft Routing
  - Aircraft Maintenance Planning
- 2 Single Day Aircraft Maintenance Routing
  - Problem Description
  - Mathematical Model
- 3 Computational Results
  - Data and Parameters
  - Numerical Results
- 4 Conclusions

# Airline Planning Process

## Airline Planning

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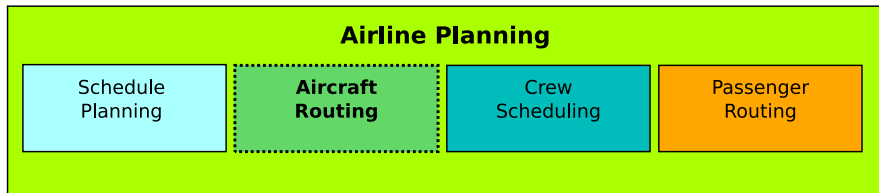
Schedule  
Planning

Aircraft  
Routing

Crew  
Scheduling

Passenger  
Routing

# Airline Planning Process



# Airline Planning Process

## Aircraft Routing

Aircraft  
Routing

Maintenance  
Planning

Tail  
Assignment



# Airline Planning Process

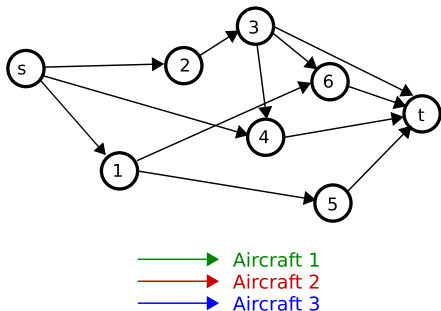
## Aircraft Routing

Aircraft  
Routing

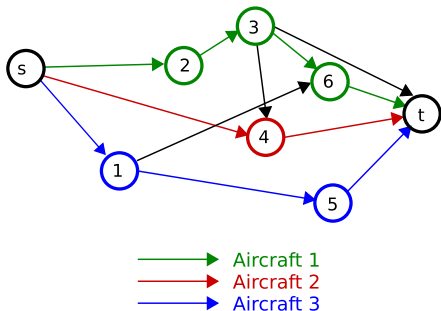
Maintenance  
Planning

Tail  
Assignment

# Aircraft routing problem



# Aircraft routing problem



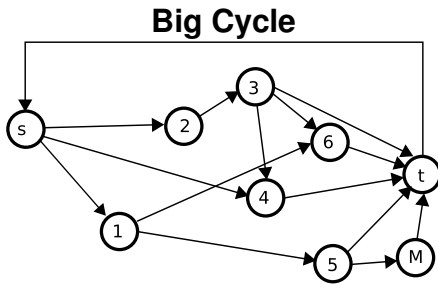
# Aircraft maintenance planning

- Regulatory requirement for airlines - aircraft must receive maintenance at regular intervals.
- Too much planned maintenance is costly and too little may violate requirements.
- Many different strategies have been developed to achieve optimal planning.
  - Flight strings, Big-cycle and One-day routes.

# Aircraft maintenance planning

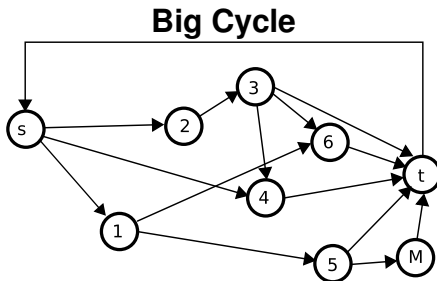
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# Maintenance planning approaches



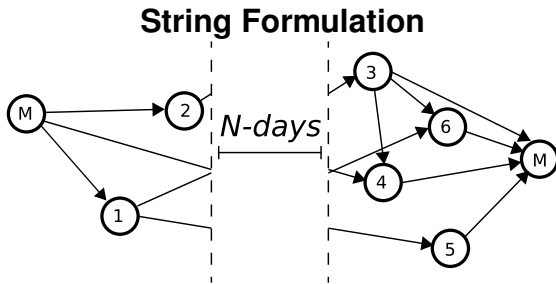
Feo and Bard (1989), Clarke et. al. (1997), Gopalan and Talluri (1998)

# Maintenance planning approaches



**s → 2 → 3 → 6 → (t, s) → 4 → (t, s) → 1 → 5 → M → t**

# Maintenance planning approaches

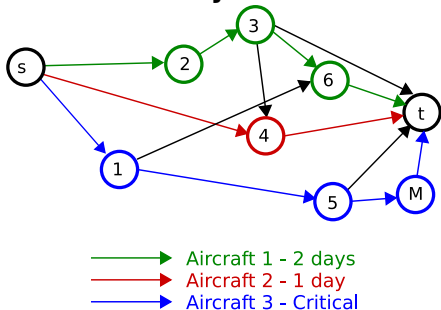


Barnhart et. al. (1998), Sriram and Haghani (2003)



# Maintenance planning approaches

## One-day Routes



Heinhold (2008), Lapp and Cohn (2012)

# Why One-day Routes?

- Big-cycle and String span multiple days.
  - Susceptible to schedule disruptions → maintenance plan becomes infeasible.
- One-day routes can adjust for schedule perturbations from previous days.
  - Ensures maintenance critical aircraft receive maintenance each day.
  - Maintenance plan is more robust day-to-day.

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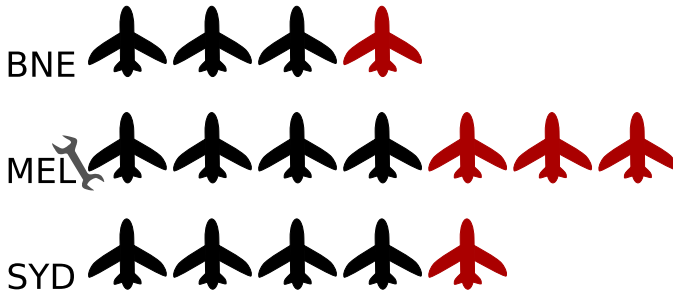
# Problem Description



## Single Day Aircraft Maintenance Routing Problem

For a single day of flying, solve an aircraft routing problem such that,

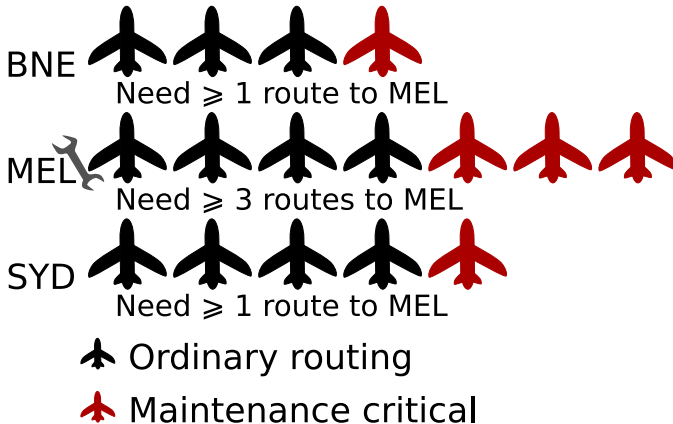
- every flight in the schedule is assigned an aircraft, and
- all maintenance critical aircraft receive maintenance that night.

# Maintenance Misalignment

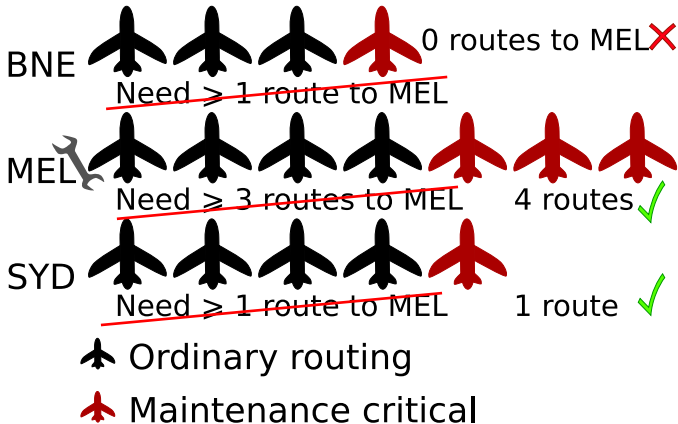


-  Ordinary routing
-  Maintenance critical

# Maintenance Misalignment



# Maintenance Misalignment



# Single Day Maintenance Planning Model

$$\min \sum_{b \in B} \sum_{p \in P_b} c_p y_p + \sum_{b \in B} F_b(M_b) \quad (1)$$

$$\text{s.t.} \quad \sum_{b \in B} \sum_{p \in P_b} a_{jp} y_p = 1 \quad \forall j \in N \quad (2)$$

$$\sum_{p \in P_b} y_p \leq R_b \quad \forall b \in B \quad (3)$$

$$M_b = \sum_{p \in P_b} o_p y_p \quad \forall b \in B \quad (4)$$

$$y_p^r \in \{0, 1\} \quad \forall b \in B, \forall p \in P_b \quad (5)$$

$$M_b \geq 0 \quad \forall b \in B \quad (6)$$



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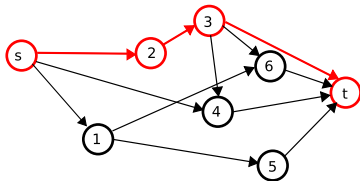
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## Solved by column generation

- Connection constraints (aircraft must fly a connected route).



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- Expected number of maintenance critical aircraft given by a binomial distribution.
- *Maintenance misalignments* are penalised by  $F_b(M_b)$ .
- $M_b$  counts the number of routes terminating at a maintenance station.
- $F_b(M_b)$  is not a linear function - causing modelling difficulties.

# Problem Reformulation

Introduce a new set of binary variables

$$\xi_b^i \in \{0, 1\}, \forall b \in B, i = 1, \dots, R_b. \quad (7)$$

Define the maintenance count variables

$$M_b = \sum_{i=1}^{R_b} \xi_b^i. \quad (8)$$

Replace  $F_b(M_b)$  in objective (1) with

$$F_b(0) - \sum_{i=1}^{R_b} \Delta_b^i \xi_b^i. \quad (9)$$

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# Airline Flight Schedule Data

- 4 different airlines with different fleet sizes and maintenance setups.
- Diverse data provides a thorough evaluation of the model.

	F267_A49	F578_A153	F1165_A289	F3370_A526
Flights	267	578	1165	3370
Aircraft	49	153	289	526
Airports	20	50	97	73
Overnight bases	12	41	67	73
Maintenance bases	1	2	5	10

Table : Flight schedule details.

## Comparison with Aircraft Routing Problem

- Standard aircraft routing solution requires modifications *a posteriori* to satisfy maintenance requirements.
- The SDAMRP aims to reduce the number of modifications that are required.

Number of maintenance misalignments at base  $b$  is given by

$$MM_b = \max \left\{ 0, \frac{R_b}{6} - \sum_{i=1}^{R_b} \xi_b^i \right\}. \quad (10)$$

The total number of maintenance misalignments is given by  $\sum_{b \in B} MM_b$

# Maintenance misalignments

<i>Misalignment</i>	F267_A49	F578_A153	F1165_A289	F3370_A526
Aircraft Routing	2.67	3.5	6.67	3.0
SDAMRP	1.17	0.5	0.0	0.0
Improvement (%)	56.25	85.71	100.0	100.0

**Table :** Maintenance misalignments for different flight schedules.

# Problem runtime

<i>Runtime</i>	F267_A49	F578_A153	F1165_A289	F3370_A526
ARP - Runtime (sec)	1.61	6.4	155.82	67675.0
SDAMRP - Runtime (sec)	7.27	6.97	168.84	65087.0

**Table :** Runtime of the ARP and SDAMRP for different flight schedules.



# Conclusions

- Developed a one-day routes approach attempting to reduce the susceptibility of maintenance planning to disruptions.
- Employed a novel modelling approach to address a complex penalty function.
- Demonstrated an effective method to reduce maintenance misalignments.